DOI: https://doi.org/10.33216/1998-7927-2021-269-5-24-28

УДК 62-83

MATHEMATICAL MODEL OF AN ASYNCHRONOUS MACHINE IN REAL COORDINATES OF STATE

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МАТЕМАТИЧНА МОДЕЛЬ АСИНХРОННОЇ МАШИНИ У РЕАЛЬНИХ КООРДИНАТАХ СТАНУ

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A mathematical description of an asynchronous machine with unreduced parameters in the space of real phase coordinates is given. When studying an asynchronous motor in a dual power supply system with controlled converters in rotor and stator circuits, there is a need for an AM model without parameters, in which the processes in the stator and rotor circuits will correspond to reality in magnitude. In addition, such a model is necessary when analyzing the energy parameters of the whole electric drive controlled by the rotor. To observe real processes in the stator and rotor, the model should be designed in separate spatial coordinate systems. In reference books for AM with a wound rotor (WR), as a rule, the real parameters (resistances, currents and voltages) of the stator and rotor and the voltage reduction coefficient (k_e) are given. Bringing currents and resistances (inductances) is carried out by coefficients $k_i = k_e$ and $k_r = k_e^2$ respectively. Let's consider the description of the machine without reduction of parameters. For the convenience of further consideration, let's introduce the general value of the mutual inductance.

We obtain an equation for the electromagnetic moment in real coordinates. Let's design a model in the MATLAB dynamic modeling environment using vector-matrix representation. Matrix algebraic operations with vector variables are implemented by Matlab Fn blocks, which are the calls to user-defined functions described in the form of M-files. The content of Matlab Fn functions is considered. On the model, the processes of starting of an AM with a phase rotor of AK-52-6 type were. Shows the graphs of start-up transients. The processes in the obtained model coincided with the results of modeling of this engine in the model with the coordinates reduced to the rotor. Thus, the energy processes described by this model correspond to the processes to the model with the given parameters, and the processes of currents and flux linkages changing of the stator and rotor are real.

The model designed in the MATLAB/Simulink dynamic modeling environment can be used to study double-powered asynchronous electric drives.

Keywords: *mathematical model, asynchronous machine, starting process, stator, rotor, given parameters.*

Introduction. When constructing mathematical models of asynchronous machines (AM), the generally accepted and most common is to consider the processes reduced to a stator or a rotor. In this case, the model uses the parameters reduced, respectively, to the stator or rotor. In addition, to simplify the mathematical description, the stator and rotor processes are considered in a single spatial coordinate system (including a triaxial one) [1]. This eliminates variable mutual inductances that depend on the angle of rotor position.

When studying electric drives (ED) controlled by a stator, it is advisable to use models with a fixed generalized coordinate system, and parameters reduced to the stator; when rotor control – with the coordinate system associated with rotor and parameters reduced to the rotor.

However, when studying an asynchronous motor in a dual power supply system with controlled converters in rotor and stator circuits, there is a need for an AM model without parameters, in which the processes in the stator and rotor circuits will correspond to reality in magnitude. In addition, such a model is necessary when analyzing the energy parameters of the whole electric drive controlled by the rotor. To observe real processes in the stator and rotor, the model should be designed in separate spatial coordinate systems. At present, software packages allow to design models of any complexity; therefore, the AM model in separate spatial coordinate systems is quite realizable.

The objective of this work is to design a mathematical model of AM with real processes in the rotor and stator, both by level and in frequency.

Research results. The electrical part of the AM with separate triaxial spatial coordinates, using the reduced (to stator or rotor) parameters, is described by the following equations

$$\mathbf{u_S} = R_S\mathbf{i_S} + \frac{d}{dt}\boldsymbol{\psi}_\mathbf{S};\ \mathbf{u_R} = R_R\mathbf{i_R} + \frac{d}{dt}\boldsymbol{\psi}_\mathbf{R}, \qquad (1)$$

where $\mathbf{u_S} = \begin{bmatrix} U_A & U_B & U_C \end{bmatrix}^T$; $\mathbf{u_R} = \begin{bmatrix} U_X & U_Y & U_Z \end{bmatrix}^T$; $\mathbf{i_S} = \begin{bmatrix} I_A & I_B & I_C \end{bmatrix}^T$; $\mathbf{i_R} = \begin{bmatrix} I_X & I_Y & I_Z \end{bmatrix}^T$; $\boldsymbol{\psi}_\mathbf{S} = \begin{bmatrix} \Psi_A & \Psi_B & \Psi_C \end{bmatrix}^T$; $\boldsymbol{\psi}_\mathbf{R} = \begin{bmatrix} \Psi_X & \Psi_Y & \Psi_Z \end{bmatrix}^T$ – vectors of voltages, currents and flux linkages of the stator and rotor in oblique coordinate systems associated with the stator (A, B, C) and the rotor (x, y, z), respectively.

Flux linkages are defined as follows

$$\begin{bmatrix} \boldsymbol{\psi}_\mathbf{S} \\ \boldsymbol{\psi}_\mathbf{R} \end{bmatrix} = \begin{bmatrix} \mathbf{L_S} & \mathbf{M_C} \\ \mathbf{M_C^T} & \mathbf{L_R} \end{bmatrix} \begin{bmatrix} \mathbf{i_S} \\ \mathbf{i_R} \end{bmatrix} = \mathbf{L} \begin{bmatrix} \mathbf{i_S} \\ \mathbf{i_R} \end{bmatrix} \qquad (2)$$

where $\mathbf{L_S} = diag(L_S\ L_S\ L_S)$; $\mathbf{L_R} = diag(L_R\ L_R\ L_R)$;

$$L_S = L_m + L_{\sigma s};\ L_R = L_m + L_{\sigma r};$$

$$\mathbf{M_C} = M_{12}\mathbf{C},\ \mathbf{C} = \begin{bmatrix} c1 & c2 & c3 \\ c3 & c1 & c2 \\ c2 & c3 & c1 \end{bmatrix},$$

$c1 = \cos\gamma_e$; $c2 = \cos(\gamma_e + 2\pi/3)$; $c3 = \cos(\gamma_e - 2\pi/3)$, where M_{12} – is the maximum value of the mutual inductance between the stator and rotor windings, $M_{12} = \frac{2}{3}L_m$;

L_m – inductance of magnetizing;

γ_e – electric angle of rotation of the motor shaft $\gamma_e = \int \omega_e dt = Z_p \int \omega dt$.

The electromagnetic moment developed by the asynchronous motor is determined from the stator and rotor currents

$$M_e = -Z_p \mathbf{i_S}^T \mathbf{M_S} \mathbf{i_R}, \qquad (3)$$

where $\mathbf{M_S} = M_{12}\mathbf{S}$, $\mathbf{S} = \begin{bmatrix} s1 & s2 & s3 \\ s3 & s1 & s2 \\ s2 & s3 & s1 \end{bmatrix}$,

$s1 = \sin\gamma_e$; $s2 = \sin(\gamma_e + 2\pi/3)$; $s3 = \sin(\gamma_e - 2\pi/3)$.

In reference books for AM with a wound rotor (WR), as a rule, the real parameters (resistances, currents and voltages) of the stator and rotor and the voltage reduction coefficient (k_e) are given. Bringing currents and resistances (inductances) is carried out by coefficients $k_i = k_e$ and $k_r = k_e^2$ respectively.

Let's consider the description of the machine without reduction of parameters. Further, the upper symbols will indicate the reduction: s – to the stator, r – to the rotor.

The system of equations (1) will have the same form

$$\mathbf{u}_\mathbf{S}^s = R_S^s\mathbf{i}_\mathbf{S}^s + \frac{d}{dt}\boldsymbol{\psi}_\mathbf{S}^s;\quad \mathbf{u}_\mathbf{R}^r = R_R^r\mathbf{i}_\mathbf{R}^r + \frac{d}{dt}\boldsymbol{\psi}_\mathbf{R}^r. \qquad (4)$$

Knowing that inductances are given in the same way as resistances, we can write

$$M_{12}^s = M_{12}^r \cdot k_r.$$

For the convenience of further consideration, let's introduce the general value of the mutual inductance

$$M_{12}^0 = \frac{M_{12}^s}{k_i} = \frac{M_{12}^s}{\sqrt{k_r}} = M_{12}^r\sqrt{k_r} = M_{12}^r k_i.$$

Let's change the components of equations (2) into real coordinates of the state

$$\boldsymbol{\psi}_\mathbf{S}^s = \mathbf{L}_\mathbf{S}^s\mathbf{i}_\mathbf{S}^s + \mathbf{M}_\mathbf{C}^s\mathbf{i}_\mathbf{R}^s = \mathbf{L}_\mathbf{S}^s\mathbf{i}_\mathbf{S}^s + \mathbf{M}_\mathbf{C}^s\frac{\mathbf{i}_\mathbf{R}^r}{k_i} =$$

$$= \mathbf{L}_\mathbf{S}^s\mathbf{i}_\mathbf{S}^s + \frac{\mathbf{M}_\mathbf{C}^s}{k_i}\mathbf{i}_\mathbf{R}^r = \mathbf{L}_\mathbf{S}^s\mathbf{i}_\mathbf{S}^s + \mathbf{M}_\mathbf{C}^0\mathbf{i}_\mathbf{R}^r, \qquad (5)$$

$$\boldsymbol{\psi}_\mathbf{R}^r = \mathbf{L}_\mathbf{R}^r\mathbf{i}_\mathbf{R}^r + \mathbf{M}_\mathbf{C}^{rT}\mathbf{i}_\mathbf{S}^r = \mathbf{L}_\mathbf{R}^r\mathbf{i}_\mathbf{R}^r + \mathbf{M}_\mathbf{C}^{rT}\mathbf{i}_\mathbf{S}^s k_i =$$

$$= \mathbf{L}_\mathbf{R}^r\mathbf{i}_\mathbf{R}^r + \mathbf{M}_\mathbf{C}^{rT}k_i\mathbf{i}_\mathbf{S}^s = \mathbf{L}_\mathbf{R}^r\mathbf{i}_\mathbf{R}^r + \mathbf{M}_\mathbf{C}^{0T}\mathbf{i}_\mathbf{S}^s$$

where $\mathbf{M}_\mathbf{C}^0 = M_{12}^0\mathbf{C}$.

Arguing in a similar way, we obtain an equation for the electromagnetic moment in real coordinates:

$$M_e = -Z_p \mathbf{i}_\mathbf{S}^{sT}\mathbf{M}_\mathbf{S}^0\mathbf{i}_\mathbf{R}^r, \qquad (6)$$

where $\mathbf{M}_\mathbf{S}^0 = M_{12}^0\mathbf{S}$.

The rotor speed is defined as the solution to motion equation

$$M_e - M_c = J\frac{d\omega}{dt}. \qquad (7)$$

Equations (4-7) represent a mathematical model of AM with real parameters.

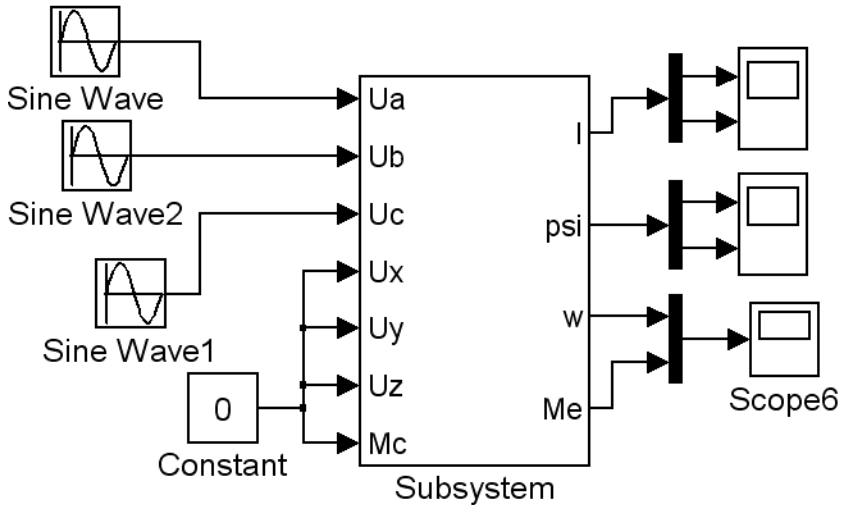
Let's design a model in the MATLAB dynamic modeling environment using vector-matrix representation [2,3]. Matrix algebraic operations with vector variables are implemented by Matlab Fn blocks, which are the calls to user-defined functions described in the form of M-files. The model in phase coordinates « $I_S^s - I_R^r$ » is shown in Figure 1.

Here functions of Matlab Fn have the following content:

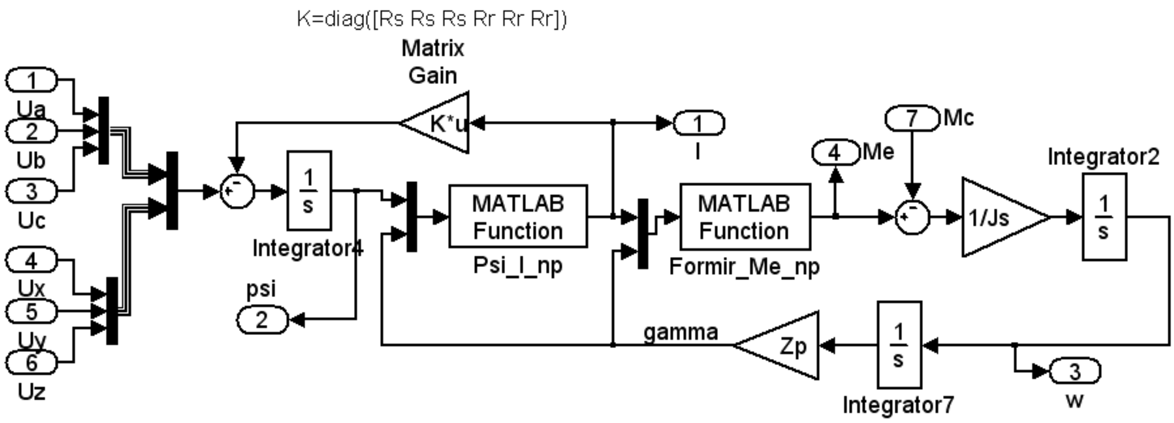
1) Matlab Fn (Psi_I_np) forms from flux linkage vector $\begin{bmatrix} \boldsymbol{\psi}_\mathbf{S} & \boldsymbol{\psi}_\mathbf{R} \end{bmatrix}^T$ and the angular position of the rotor, according to the equation

$$\begin{bmatrix} \mathbf{i_S} \\ \mathbf{i_R} \end{bmatrix} = \mathbf{L}^{-1} \begin{bmatrix} \boldsymbol{\psi}_\mathbf{S} \\ \boldsymbol{\psi}_\mathbf{R} \end{bmatrix};$$

2) Matlab Fn (Formir_Me_np) forms the electromagnetic moment (6) from the vector of currents and the position of the rotor shaft.



a



b

Fig. 1. AM model in the MATLAB dynamic modeling environment:
a – conection of the block-subsystem of the AM model; b – the structure of the subsystem

Table

Matlab Fn function content

Matlab Fn (Psi_I_np)	Matlab Fn (Formir_Me_np)

function Io=Psi_I(u)
global Ls Lr M12_0
Psio=u(1:6)';
Gamma=u(7);
c1=M12_0*cos(Gamma);
c2=M12_0*cos(Gamma+2*pi/3);
c3=M12_0*cos(Gamma+4*pi/3);
Lo=[Ls 0 0 c1 c2 c3;
0 Ls 0 c3 c1 c2;
0 0 Ls c2 c3 c1;
c1 c3 c2 Lr 0 0;
c2 c1 c3 0 Lr 0;
c3 c2 c1 0 0 Lr];
Psio=Psio(:);
Io=inv(Lo)*Psio;
Io=[Io'];
	function
	Mem=Formir_Me(u)
	global Zp M12_0
	Is=u(1:3)';
	Ir=u(4:6);
	Gamma=u(7);
	m1=M12_0*sin(Gamma);
	m2=M12_0*sin(Gamma+
	2*pi/3);
	m3=M12_0*sin(Gamma+
	4*pi/3);
	MM=[m1 m2 m3;
	m3 m1 m2;
	m2 m3 m1];
	Mem=-Zp*Is*MM*Ir;

On the model, the processes of starting of an AM with a phase rotor of AK-52-6 type with the following parameters were investigated

$U_{1H}=380$ V; $E_{2H}=85$ V; $I_{1H}=8$ A; $I_{2H}=21,2$ A;

$k_r=k_e^2=18$;

$J=0,1\,\text{kg}\cdot\text{m}^2$; $n_H=910$ rpm; $R_S^s=1,23$ Ω;

$R_R^r=0,15$ Ω;

$x_m^r=5,5$ Ω; $x_1^r=0,3$ Ω; $x_2^r=0,18$ Ω.

As a result of the calculation:

$L_S^s=0,332$ H; $L_R^r=0,0181$ H; $M_{12}^0=0,0495$ H.

Figure 2 shows the graphs of start-up transients. The processes $M_e(t)$ and $\omega(t)$ in the obtained model coincided with the results of modeling of this engine in the model with the coordinates reduced to the rotor [4, 5].

Thus, the energy processes described by this model correspond to the processes to the model with the given parameters, and the processes of currents and flux linkages changing of the stator and rotor are real.

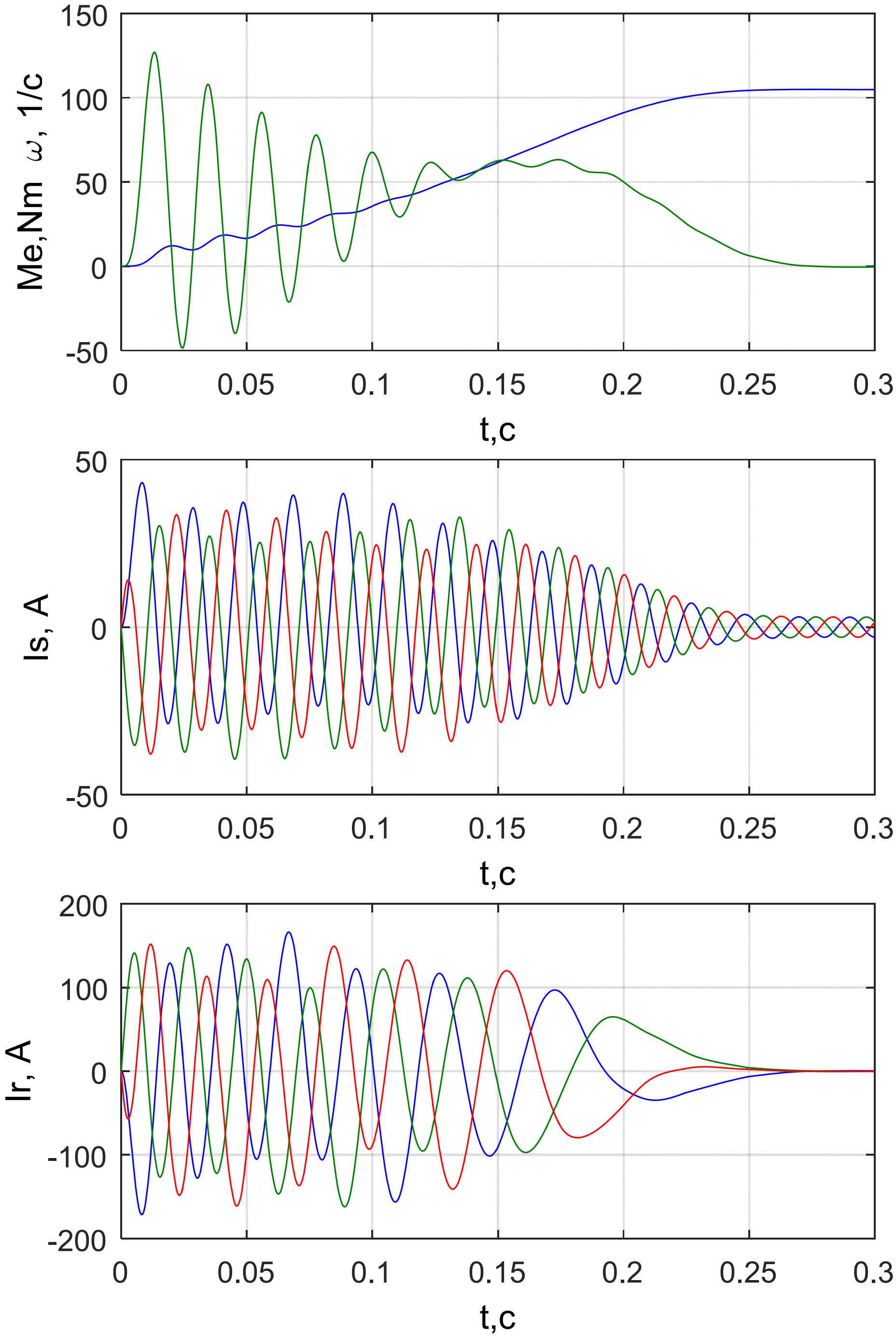


Fig. 2. Transient processes during starting the AK-52-6 machine

Conclusions. A mathematical description of an asynchronous machine in the space of real phase coordinates with unreduced parameters is obtained.

To take into account the mutual magnetic influence of the stator and the rotor, the concept of general mutual inductance is proposed.

Modeling has confirmed the correctness of the accepted approach.

The resulting model can be used in the study of asynchronous electric drives designed by system of a dual power supply.

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Руднєв Є.С., Шевченко І.С., Романченко Ю.А. Математична модель асинхронної машини у реальних координатах стану

Наведено математичний опис асинхронної машини з неприведеними параметрами в просторі реальних фазових координат. У довідниках для АМ з фазним ротором (ФР), як правило, наводяться реальні параметри (опору, струми і напруги) статора і ротора і коефіцієнт приведення напруг. Приведення струмів і опорів (індуктивностей) здійснюється коефіцієнтами $k_i = k_e$ *та* $k_r = k_e^2$ *відповідно. У даній статті розглянуто опис машини без приведення цих параметрів. Для зручності подальшого розгляду введено загальне значення взаємної індуктивності.*

Отримано вираз для електромагнітного моменту в реальних координатах. Побудована модель в середовищі динамічного моделювання MATLAB, використовуючи векторно-матричне представлення. Матричні алгебри з векторними змінними реалізуються блоками Matlab Fn, що представляють собою звернення до призначених для користувача функцій, описаних у вигляді М-файлів. Розглянуто зміст функцій Matlab Fn. На моделі досліджувалися процеси пуску АМ з фазним ротором типу АК-52-6. Наведено графіки перехідних процесів пуску. Процеси в отриманій моделі співпали з результатами моделювання цього двигуна в моделі з координатами, наведеними до ротора. Енергетичні процеси, описувані цією моделлю, відповідають процесам в моделі з наведеними параметрами, а процеси зміни струмів і потокозчеплення статора і ротора є реальними.

Побудована в середовищі динамічного моделювання MATLAB / Simulink модель може використовуватися при дослідженні асинхронних електроприводів подвійного живлення.

Ключові слова: *математична модель, асинхронна машина, процес пуску, статор, ротор, наведені параметри.*

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Стаття подана 05.08.2019.